

$a_1 \pi$ CONTRIBUTION TO $e^+e^- \rightarrow 4\pi$ ANNIHILATION AND $\tau \rightarrow 4\pi\nu_\tau$ DECAY

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Abstract

The results of the study of the process $e^+e^- \rightarrow 4\pi$ by the CMD-2 collaboration at VEPP-2M are presented. Analysis of the differential distributions demonstrates the dominance of the $a_1\pi$ and $\omega\pi$ intermediate states. Simple model, based on the assumption of $a_1(1260)\pi$ and $\omega\pi$ dominance as intermediate states, successfully describes also the data of CLEOII and ALEPH obtained recently for the decay $\tau \rightarrow 4\pi\nu_\tau$.

1 Introduction

Production of four pions is one of the dominant processes of e^+e^- annihilation into hadrons in the energy range from 1.05 to 2.5 GeV. Due to the conservation of vector current (CVC) the cross section of this process is related to the probability of $\tau \rightarrow 4\pi\nu_\tau$ decay [1]. Therefore, all realistic models describing first process should also be appropriate for description of another one.

One of the main difficulties in the experimental studies of four pion production is caused by the existence of different intermediate states via which the final state could be produced, such as $\omega\pi$, $\rho\sigma$, $a_1(1260)\pi$, $h_1(1170)\pi$, $\rho^+\rho^-$, $a_2(1320)\pi$, $\pi(1300)\pi$. The abundance of various possible mechanisms and their complicated interference results in the necessity of simultaneous analysis of two possible final states ($2\pi^+2\pi^-$ and $\pi^+\pi^-2\pi^0$).

In Section II of this report I present results from a model-dependent analysis of both possible channels in e^+e^- annihilation into four pions based on data collected with the CMD-2 detector in the energy range 1.05-1.38 GeV [2]. To describe four pion production a simple model was used assuming quasitwoparticle intermediate states and taking into account the important effects of the identity of the final pions as well as the interference of all possible amplitudes. It was

unambiguously demonstrated that the main contribution to the cross section of the process $e^+e^- \rightarrow 4\pi$ in the energy range 1.05 – 1.38 GeV, in addition to previously well-studied $\omega\pi^0$, is given by $\rho\pi\pi$ intermediate state. Moreover, the latter is completely saturated by the $a_1\pi$ mechanism. The contribution of other intermediate states was estimated to be less than 15 %.

In Section II I also discuss the results for $e^+e^- \rightarrow 2\pi^+2\pi^-$ cross section obtained recently by CMD-2 in the energy region 0.60–0.97 GeV [3]. In this energy region the energy dependence of the cross section agrees with the assumption of the $a_1\pi$ intermediate state which is dominant above 1 GeV.

In Section III the comparison is performed of the available experimental data for $\tau \rightarrow 4\pi\nu_\tau$ decay [4, 5, 6, 7] with the prediction of the model based on the assumption of the $a_1\pi$ and $\omega\pi$ dominance [8]. It is shown that the model successfully describes experimental data on $\tau \rightarrow 4\pi\nu_\tau$ decay.

2 The process $e^+e^- \rightarrow 4\pi$

The results described here are based on 5.8 pb⁻¹ of e^+e^- data collected at energies $2E_{beam}$ from 1.05 up to 1.38 GeV at the VEPP-2M collider with the CMD-2 detector [2].

Figures 1, 2 show the total cross sections $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ and $e^+e^- \rightarrow 2\pi^+2\pi^-$ vs energy.

The cross sections measured in [2] is consistent with the previous measurement at OLYA [9], CMD [10], and within systematic errors don't contradict to the recent result from SND [11]. For $e^+e^- \rightarrow 2\pi^+2\pi^-$ the value of the cross section from all three groups is significantly lower than that from ND [12, 13]. Above 1.4 GeV the results from Orsay [14, 15, 16] and Frascati [17, 18, 19] groups are shown.

To obtain the $a_1\pi$ contribution to the total $\pi^+\pi^-2\pi^0$ cross section, the contribution of the $\omega\pi$ intermediate state was subtracted. Such a procedure is possible because the interference between $\omega\pi$ and $a_1\pi$ is numerically small ($\sim 5\%$) due to the small width of ω meson. Figure 3 presents the ratio of the cross sections $\sigma(e^+e^- \rightarrow 2\pi^+2\pi^-)$ and $\sigma(e^+e^- \rightarrow \pi^+\pi^-2\pi^0)$ where the contribution of $\omega\pi^0$ is subtracted. The solid curve shows the theoretical prediction based on the $a_1\pi$ dominance.

Figure 4 shows distributions over $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^\pm\pi^\pm)$, $M_{recoil}(\pi^\pm)$ and $\cos(\psi_{\pi^+\pi^-})$ for $2\pi^+2\pi^-$ case. One can see that the hypothesis of the $a_1\pi$ dominance is in agreement with the data.

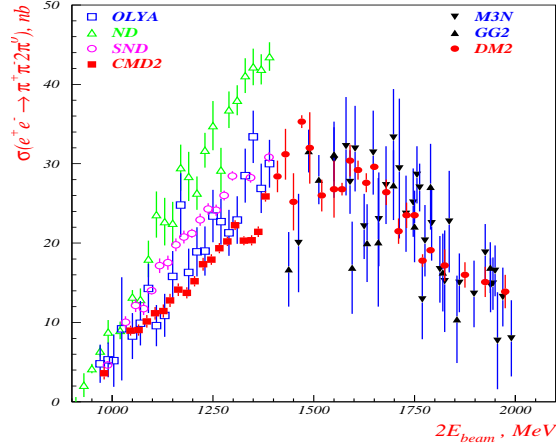


Figure 1: Energy dependence of the $\pi^+\pi^-2\pi^0$ cross section

For the analysis of $\pi^+\pi^-2\pi^0$ channel the data sample was subdivided into two classes: 1. $\min(|M_{recoil}(\pi^0) - M_\omega|) < 70 \text{ MeV}$, 2. $\min(|M_{recoil}(\pi^0) - M_\omega|) > 70 \text{ MeV}$, where M_ω is the ω mass. The first class contains mostly $\omega\pi$ events while their admixture in the second class is relatively small, about $(1 \div 5)\%$ depending on the beam energy. In [2] it was shown that the process $\pi^+\pi^-2\pi^0$ is well described in the minimal model in which there are two intermediate states $\omega\pi$ and $a_1\pi$ only. Similar consistence is observed at other energies.

Recently, using 3.07 pb^{-1} of data collected in the energy range 0.60–0.97 GeV by CMD-2, about 150 events of the process $e^+e^- \rightarrow 2\pi^+2\pi^-$ have been selected [3]. Figure 5 shows the energy dependence of the cross section below 1.05 GeV. For illustration, results from other measurements [10, 12, 14, 20, 21] are also demonstrated. The values of the cross section obtained in [3] are consistent with them and match the measurements of CMD-2 above the ϕ meson. The overall systematic uncertainty was estimated to be $\approx 12\%$.

The shaded area in Figure 5 corresponds to the extrapolation of the energy

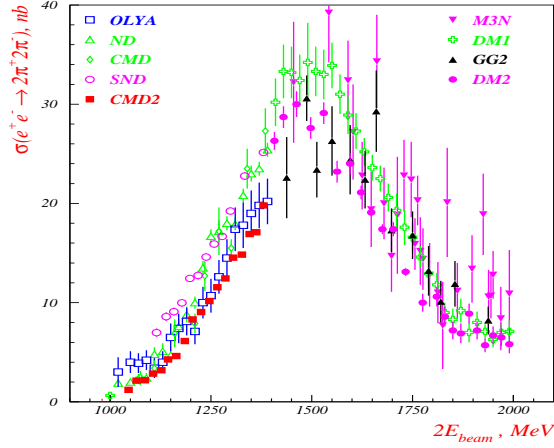


Figure 2: *Energy dependence of the $2\pi^+2\pi^-$ cross section*

dependence of the cross section from the energy region above 1.05 GeV [2]. The calculation assumed that the cross section behaviour is determined by two interfering resonances - ρ and its excitation ρ' decaying into the final four pion state via the $a_1\pi$ intermediate mechanism. The central curve corresponds to the a_1 width of 600 MeV optimal in the analysis whereas the upper and lower curves are obtained for the widths of 800 and 400 MeV respectively. It can be seen that the energy dependence of the data is consistent with the assumption of the $a_1\pi$ dominance earlier established at higher energies [2].

3 $a_1\pi$ contribution to $\tau \rightarrow 4\pi\nu_\tau$ decay

The initial hadron state which decays into four pions for both τ decays and e^+e^- annihilation has the ρ -meson quantum numbers and is referred to as $\tilde{\rho}$. Due to the conservation of vector current the probability $d\Gamma_1$ and $d\Gamma_2$ of $\tau^- \rightarrow$

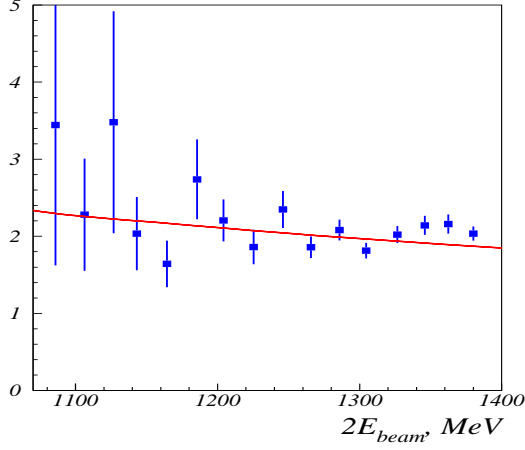


Figure 3: *Energy dependence of $\sigma(e^+e^- \rightarrow 2\pi^+2\pi^-)/\sigma(e^+e^- \rightarrow \pi^+\pi^-2\pi^0)$. The contribution of $\omega\pi^0$ is subtracted. The solid curve shows the theoretical prediction based on the $a_1\pi$ dominance*

$\pi^-\pi^+\pi^-\pi^0\nu_\tau$ and $\tau^- \rightarrow \pi^-\pi^0\pi^0\pi^0\nu_\tau$ decays respectively can be written as

$$\frac{d\Gamma_i}{ds} = \frac{G^2|V_{ud}|^2}{96\pi^3m_\tau^3}(m_\tau^2 + 2s)(m_\tau^2 - s)^2 R_{4\pi} \frac{dW_i}{W_1 + W_2} \quad (1)$$

where G is the Fermi constant, $R_{4\pi}$ is the ratio of the cross section $e^+e^- \rightarrow 4\pi$ and $e^+e^- \rightarrow \mu^+\mu^-$, dW_1 and dW_2 are the probabilities of $\tilde{\rho}^-$ decays into $\pi^-\pi^+\pi^-\pi^0$ and $\pi^-\pi^0\pi^0\pi^0$, respectively. Let dW_3 and dW_4 are the probabilities of $\tilde{\rho}^0$ decays into $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$. Due to the isospin invariance, we have $W_1 = W_3/2 + W_4$ and $W_2 = W_3/2$. The explicit forms of the matrix elements, corresponding to W_i , are presented in [8]. In order to get the predictions for τ decay the interference between $\omega\pi$ amplitude and $a_1\pi$ was neglected, and (1) was written in the following form:

$$\frac{d\Gamma_1}{ds} = \frac{G^2|V_{ud}|^2}{96\pi^3m_\tau^3}(m_\tau^2 + 2s)(m_\tau^2 - s)^2 \left[R_{\omega\pi} \frac{dW_\omega}{W_\omega} + R_{2\pi^+2\pi^-} \frac{dW_1}{W_3} \right], \quad (2)$$

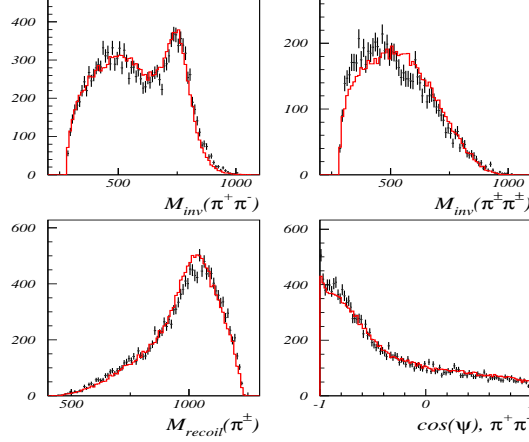


Figure 4: *Distributions over $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^\pm\pi^\pm)$, $M_{recoil}(\pi^\pm)$ and $\cos(\psi_{\pi^+\pi^-})$ for $2\pi^+2\pi^-$ events*

where dW_ω is the probability of $\tilde{\rho}^- \rightarrow \omega\pi^-$ decay, $R_{\omega\pi}$ is the ratio of the cross section $e^+e^- \rightarrow \omega\pi$ and $e^+e^- \rightarrow \mu^+\mu^-$.

In order to fix the parameters of the model the data of $e^+e^- \rightarrow 4\pi$ [2] and $\tau^- \rightarrow 2\pi^0\pi^-2\nu_\tau$ [23] were used. The mass of a_1 was taken from the PDG table [22] and the width was obtained as a result of optimal description of three pion invariant mass distribution in $\tau^- \rightarrow 2\pi^0\pi^-\nu_\tau$ decay. This value of a_1 width also provides a good description of $e^+e^- \rightarrow 4\pi$ data.

In [23] it was obtained the evidence that a_1 meson has significant probability to decay into three pions through $\sigma\pi$ intermediate state. The data analysis of $e^+e^- \rightarrow 4\pi$ also confirmed this statement. In [8] the admixture of $\sigma\pi$ to the a_1 decay amplitude was taken into account, and the parameters of this admixture were extracted from $e^+e^- \rightarrow 4\pi$ data.

The most interesting information on the mechanism of four pion channel can be obtained from two-pion mass distributions. In [8] it was shown that data of CLEOII detector [6] obtained without subtraction of $\omega\pi^-$ contribution are in

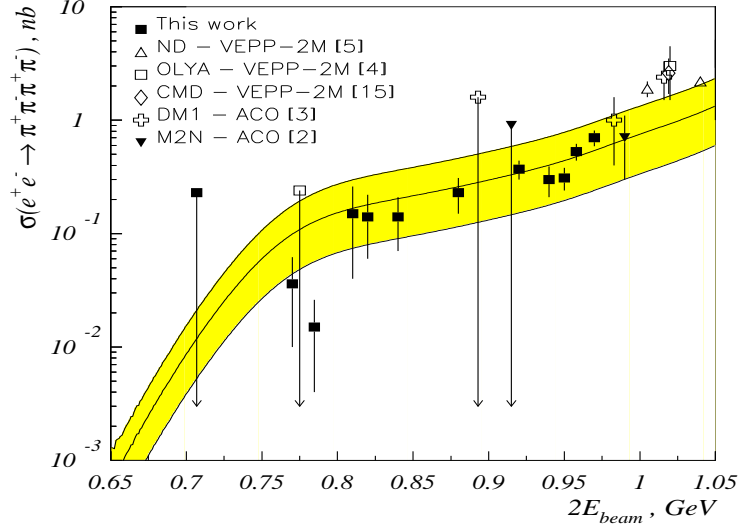


Figure 5: Cross section of the process $e^+e^- \rightarrow 2\pi^+2\pi^-$ below 1.05 GeV

good agreement with predictions. The data obtained with the subtraction of $\omega\pi^-$ contribution allows one to make a more detail comparison of the differential distributions predicted within the assumption on $a_1\pi$ dominance. For this purpose the data obtained by ALEPH [5] (see Fig. 6) and very recent high-statistics data of CLEOII [7] were used. In the first case we see a good agreement in spite of the absence of possibility to take into account the detector efficiency and energy resolution. The agreement for the data of CLEOII[7] is a little bit worse. Unfortunately, in this data the contributions of background events (such as $K^0\pi^-\pi^0\nu_\tau$ and $K^\pm\pi^\mp\pi^-\pi^0\nu_\tau$) were not subtracted, though their fraction was significant (about 8%). New analysis of $\tau \rightarrow 4\pi\nu_\tau$ by CLEOII [24] completely confirmed conclusion on the production mechanism based on the assumption of a_1 dominance.

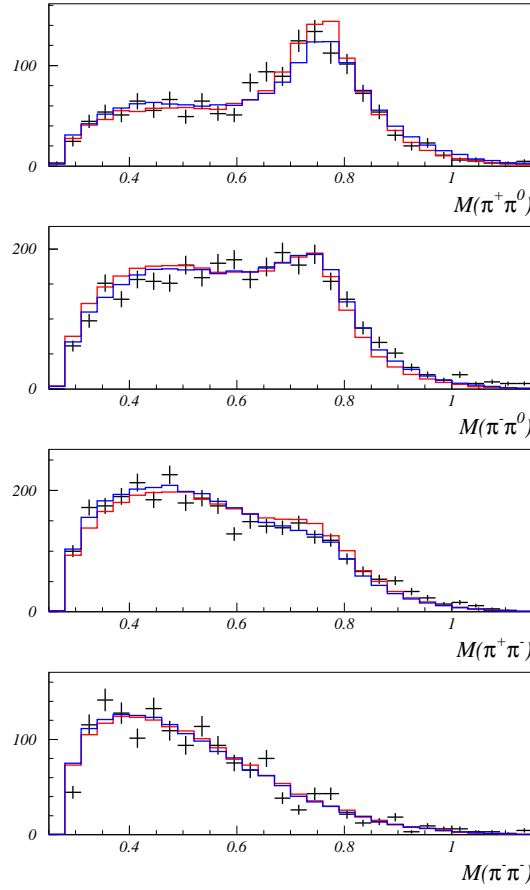


Figure 6: *Two-pion invariant mass distributions for $\tau^- \rightarrow 2\pi^-\pi^+\pi^0\nu_\tau$ decay after $\omega\pi^-$ events subtraction, obtained by ALEPH.*

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